APPLICATION OF INTERLEAVING MODELS TO DESCRIBE INTRUSIVE LAYERS IN THE DEEP POLAR WATER OF THE ARCTIC BASIN



(triangles) and AO-94 (criss-crosses) CTD data used in this work.

Introduction

The analysis of the hydrological parameters of the frontal zones based on the large amount of CTD data presented in [1, 2] demonstrated that fronts are in abundance in the upper part of the Deep Polar Water layer (DPW) in the Eurasian Basin, and DPW can be characterized by intense interleaving layering and different thermohaline structures. Unlike the intrusive layering of the upper layer of the Eurasian Basin, which is usually found under the conditions of unstable temperature or salinity stratification, the intense intrusive layering in the DPW layer in the depth range of ~600–1300 m is observed under conditions of absolutely stable stratification [2]. It is assumed that differential mixing (i.e. vertical heat exchange, which is of the close order of magnitude as molecular diffusivity) is the main mechanism of the intrusion formation. This paper is dedicated to the calculation of all the important interleaving parameters on the basis of instability models of pure thermohaline and baroclinic frontal zones and their comparison with the field results of the front observations, which were described in detail in [1, 2].

Field data

The CTD observations used were obtained in the Nansen, Amundsen basins and over the Lomonosov Ridge from IB Oden in 1991 (hereafter Oden-91), R/V Louis St. Laurent in 1994 (AO-94) and R/V Polarstern in 1996 (PS-96) (Fig. 1). The instruments used were Neil Brown MarkIIIb CTDs and occasionally Sea-bird SBE-11. Fig. 1 gives positions of all CTD casts used in the analysis. Detailed descriptions of the data are given, for example, in *Carmack et* al.[1997], Rudels et al. [1999].

For the description of intrusive layering at the fronts considered below, a 3D model of pure thermohaline front instability and a 2D model of baroclinic front instability with the parametrization of differential mixing were used [3, 4].



Fig. 2. Mean thermohaline fields (at the right side) and corresponding successive CTD-profiles (at the left side; the Oden-91 transect, stations 27-38).

Calculations of the parameters of the intrusive layering for a pure thermohaline front

Recording to the empirical analysis [1], the mean hydrological parameters of the mont over the entire ray
$\overline{T}_{x} = 1.7 \cdot 10^{-7} {}^{o}\mathrm{C}/\mathrm{m}, N = 0.0015 \mathrm{s}^{-1}, \ \overline{\varepsilon}_{z} = \frac{g}{2}$
where \overline{T}_x is the mean horizontal temperature gradient, N is the mean Brunt–Väisälä frequency, $\overline{\varepsilon}_z$ is the n
emperature gradient, and g is the acceleration due to gravity. The width of the front was assumed to be e

and $\overline{T}_x = 4 \cdot 10^{-7} \, ^{\circ} \text{C/m}$ (supposed value of the mean frontal temperature gradient before the appearance of instability) are given in **Table 1** [4].

$\overline{T}_{\rm r} = 1.7 \cdot 10^{-7} \, {}^{\circ}\,{\rm C}\,/\,{\rm m}$

$k^*, \mathbf{m}^2/\mathbf{s}$	0.0	$0.5 \cdot 10^{-7}$	10 ⁻⁷	$1.5 \cdot 10^{-7}$	$2 \cdot 10^{-7}$	$3 \cdot 10^{-7}$
$\lambda \cdot 10^9$, s ⁻¹	8.8	6	4.7	4	3.4	2.7
$T = 1/\lambda$, yrs	3.6	5.3	6.7	7.9	9.3	11.7
m, \mathbf{m}^{-1}	0.3	0.18	0.14	0.11	0.1	0.07
$h = \pi/m, \mathbf{m}$	9.9	17.2	22.7	28.2	33.0	42.7

 $\overline{T}_{\rm x} = 4 \cdot 10^{-7} \, {}^{\circ} \, {\rm C} \, / \, {\rm m}$

$k^*, \mathbf{m}^2/\mathbf{s}$	0.0	$0.5 \cdot 10^{-7}$	10-7	$1.5 \cdot 10^{-7}$	$2 \cdot 10^{-7}$	$3 \cdot 10^{-7}$	The
$\lambda \cdot 10^8$, s ⁻¹	2.1	1.4	1.1	0.9	0.8	0.6	verti
$T = 1/\lambda$, yrs	1.5	2.3	2.9	3.5	4.0	5.3	this c
m, \mathbf{m}^{-1}	0.48	0.28	0.21	0.17	0.15	0.11	verti
$h = \pi/m, \mathbf{m}$	6.5	11.1	15	18.2	21.4	27.6	(Tab
							diffe

Table 1. Results of calculations of intrusion layering parameters for a pure thermohaline front.

Thus, the interleaving model for a pure thermohaline front with parameterization of the differential mixing describes quite well the slopes and vertical scales of the intrusions.

The vertical scale of the most unstable mode was determined as a half-wave of the sinusoidal perturbation along the vertical coordinate z. It is approximately equal to 20-25 meters. It is seen from **Table 1** that, for $\overline{T}_{r} = 1.7 \cdot 10^{-7} \circ C / m$ and $k^* \times 10^7 m^{-2} s = 0.5, 1, 1.5, and 2$ the calculated thickness of the most unstable mode differs by not greater than 33% of the mean thickness of the observed intrusions. At the mean horizontal gradient $\overline{T}_{r} = 4 \cdot 10^{-7} \,^{\circ} \text{C} \,/\,\text{m}$ satisfactory agreement between the calculated and measured thicknesses of the intrusions is observed at $k^* \times 10^7$ m⁻² s = 1.5, 2, and 3

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Example of a pure thermohaline front (empirical analysis of intrusive layers brief results) Fig. 2a shows a sequence of salinity, temperature and density profiles from the 27–38 stations of the Oden-91 transect in the depth range of 800–1050 m. Corresponding mean salinity, temperature and density contours (obtained by low-pass cosine filtering) versus distance and depth for stations 27–38 are given in Fig. 2b. At stations 29-38 intensive, coherent intrusive layers are observed having clearly expressed sawtooth shape in the salinity and temperature profiles. The thickness of high-gradient layers, where temperature and salinity increase with depth, is 10–20 m, while the low-gradient layers, with temperature and salinity decreasing with

Fig. 3. θ , S-diagrams for stations 34-32 (Oden-91).

The estimates of the hydrological parameters performed in [1] demonstrated that the front found at stations 32–34 is a practically pure thermohaline front (the baroclinicity is low in the frontal zone). It was also found in this work on the basis of the analysis of T,S diagrams (Fig. 3) that warm and saline (fresh and cold) intrusions cross the isopycnals at small angles and descend (ascend) relative to the isopycnals according to the assumption about the mechanism of differential mixing.

According to the empirical analysis [1], the mean hydrological parameters of the front over the entire layer involved in the layering were equal to the following: $\frac{g\alpha T_z}{m^2} \approx 0.6$

> mean parameter characterizing the temperature stratification, T_z is the mean vertical equal to 100000 m.

The growth rate λ , the vertical wavenumbers *m*, and the vertical scales of the most unstable modes *h* calculated at different coefficients of turbulent mixing at $\overline{T}_x = 1.7 \cdot 10^{-7} \circ \text{C} / \text{m}$ (empirical analysis)

empirical estimate of the intrusion slope to the horizontal plane for the calculation of the ical turbulent diffusivity based on the theory in [2], which was used in [1], demonstrated that coefficient should be equal to $k^* \approx 1.5 \times 10^{-7} \text{ m}^2/\text{s}$. At this value of the diffusivity, the ical scale of the most unstable mode satisfactorily corresponds to the thickness of the intrusions ble 1). Thus, the interleaving model for a pure thermohaline front with parameterization of the differential mixing describes quite well the slopes and vertical scales of the intrusions.



Pr	1	1000	3000	3310	$4 \cdot 10^{3}$	104	105
λ, s^{-1}	/////	/////	/////	$1.1 \cdot 10^{-13}$	$6.8 \cdot 10^{-10}$	4.9.10-9	5.9·10 ⁻⁹
$T = 1/\lambda$, yrs		/////	/////	2·10 ⁵	46	6.5	5.4
$h = \pi/m$, m		/////	/////	12.8	5.23	5.2	11.8
θ	/////	/////	/////	0.0025	0.0024	0.0018	0.0009
$Ri^* = (Pr+1)^2/4Pr$	/////	/////	/////	828	10 ³	$2.5 \cdot 10^{3}$	$2.5 \cdot 10^4$

$\varepsilon_z = 0.7$			$\mathcal{E}_z = 0.9$		
$k^*, m^2/s$	$0.3 \cdot 10^{-7}$	$0.3 \cdot 10^{-8}$	$k^*, m^2/s$	$0.3 \cdot 10^{-7}$	$0.3 \cdot 10^{-8}$
λ, s^{-1}	$8.6 \cdot 10^{-11}$	$3.4 \cdot 10^{-10}$	λ , s ⁻¹	$5 \cdot 10^{-10}$	$1.9 \cdot 10^{-9}$
$T = 1/\lambda$, yrs	368	93	$T = 1/\lambda$, yrs	63	16.7
$h = \pi/m$,m	20.5	7.1	$h = \pi/m, m$	17.9	5.2
θ	$4.5 \cdot 10^{-4}$	$6.1 \cdot 10^{-4}$	θ	$6 \cdot 10^{-4}$	$13 \cdot 10^{-4}$

$\varepsilon_{\gamma} = 0.95, N = 0.0025 \mathrm{s}^{-1}, \gamma_{\rho} = 0.7 \cdot 10^{-3},$ $Ri = 6.4 \cdot 10^4, Ri^* = 2.5 \cdot 10^4, k^* = 0.25 \cdot 10^{-7} \mathrm{m}^2/\mathrm{s}$					
	Pr	20000	15000		
	λ, s^{-1}	$3.8 \cdot 10^{-9}$	$3.2 \cdot 10^{-9}$		
	$T = 1/\lambda$, yrs	8.3	9.9		
	$h = \pi/m, \mathrm{m}$	7.1	6.5		
	θ	$1.2 \cdot 10^{-3}$	$1.3 \cdot 10^{-3}$		

In conclusion, we would like to note that, according to our calculations using the interleaving model describing intrusion formation at a baroclinic front, the slopes of the most unstable modes satisfactorily correspond to the realistic slopes, whereas the vertical scales of them by order of magnitude are close to the scale of real intrusions only when the momentum diffusivity is high. It is difficult to say whether such a situation was ever realized in the ocean. Nevertheless, we cannot exclude that in the region of intrathermocline baroclinic eddy perturbations when linear waves are radiated, the vertical momentum diffusivity can be significantly greater that the diffusivity of the heat (mass).

Conclusion The interleaving model for a pure thermohaline front with parameterization of the differential mixing describes quite well the slopes and vertical scales of the intrusions, while interleaving model for a baroclinic front shows satisfactorily correspondence of all most unstable modes parameters and real intrusions only when the momentum diffusivity is high.

Ref	erences
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Fig. 4. Mean thermohaline fields (at the right side of the figure) and corresponding successive CTD-profiles (at the lef side; the PS-96 transect). Numbers near the profiles (see upper fragment at the left side of the illustration) designate the numbers of stations. Location of the CTD-stations are shown in the transection of the mean density field.

 $\overline{T} \approx 0.5 \cdot 10^{-6} \, {}^{\circ}\text{C} \, /\text{m} \, N = 0.0025 \, \text{S}^{-1}, Ri = 4.9 \cdot 10^3, \varepsilon_1 = 0.9999, \gamma_2 = 8 \cdot 10^{-4}, k^* = 0.25 \cdot 10^{-7} \, \text{m}^2/\text{s}$

Table 2. Results of calculations of intrusion layering parameters for a baroclinic front.

As it can be seen from **Table 2**, the model calculations demonstrate that instability appears at the minimum Prandtl number Pr = 3310. The model calculations demonstrate that instability appears at the minimum Prandtl number Pr = 3310. The Prandtl number at which instability is possible at the given hydrological parameters of the problem seems to be too high to consider that it is possible under realistic conditions. However, one should pay attention to the fact that, at such a value of the Prandtl number, the coefficient of momentum diffusivity is as follows: $k_{\text{mom}} \approx 0.8 \times 10^{-4} \text{ m}^2/\text{s}$. Thus, it is not very high if we consider that the maximum estimate of the coefficient of momentum diffusivity in the oceanic thermocline far from the coasts and bottom topography peculiarities is $k_{mom}^{max} \approx 1-5 \times 10^{-4} \text{ m}^2/\text{s}$. The vertical scale of the most unstable mode when Pr = 3310 is 13 meters; i.e., it is close to the vertical scale of intrusions at the baroclinic front, but the formation time of the most unstable mode significantly exceeds 10 years; therefore, this case is unrealizable. If Pr = 10000, the time of the most unstable mode formation is less than ten years, the vertical coefficient of momentum diffusivity value is satisfactory, but the vertical scale of the most unstable mode is smaller than the characteristic scale of intrusions observed at the baroclinic front - this case is also unrealizable.

$$T_{\rho} = 4 \cdot 10^{-4}, Ri = 3 \cdot 10^{4}, Ri^{*} = 2.5 \cdot 10^{4}, Pr = 10^{-4}$$

Table 3. Results of calculations of intrusion layering parameters for a baroclinic front
 with input parameters, calculated from the empirical data.

> $Pr = 16000, N = 0.002 \, \mathrm{s}^{-1}, \gamma_{\rho} = 1.1 \cdot 10^{-3},$ $4.1.10^4$ $Pi^* = 2.5.10^4$ $k^* = 0.12.10^{-7} m^2/r$

$R_l = 4.1 \cdot 10^{-1}, R_l = 2.5 \cdot 10^{-1}, k = 0.12 \cdot 10^{-1} \text{m}^2/\text{s}$						
\mathcal{E}_{z}	0.65	0.75				
λ , s ⁻¹	$3 \cdot 10^{-9}$	$4.7 \cdot 10^{-9}$				
$T = 1/\lambda$, yrs	10.6	16.7				
$h = \pi/m, m$	5.1	4.7				
θ	$1.3 \cdot 10^{-3}$	$1.5 \cdot 10^{-3}$				

Table 4. Results of calculations of intrusion layering parameters for a baroclinic
 front with input parameters, at which restrictions on the coefficient of momentum diffusivity and the formation time of the most unstable modes are fulfilled.

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Fig. 5. θ , S diagrams in the depth range of 800–1050 m for stations 40–42 of PS-96.

Example of a baroclinic front (empirical analysis of intrusive layers brief results) Fig. 4a shows a sequence of salinity, temperature and density profiles from the 36–50 stations of the PS-96 transect Intense intrusive layering is observed at sharp fronts in the region of absolutely stable stratification (800–1050 m). The vertical scale of highgradient layers (in which temperature and salinity increase with depth) is 10 m, of lowgradient layers (in which temperature and salinity decrease with depth) is 30-40 m. Mean (the filter parameter is 100 meters) and full salinity, temperature and density contours versus distance and depth for stations 36–50 are given in **Fig. 4b** and **Fig. 4c**.



The coherence of intense intrusions in the region of absolutely stable stratification is especially high at stations 40–42. According to the empirical analysis the intrusions recorded at these stations cross the isopycnals (Fig. 5); the warm and saline (cold and fresh) intrusions become more (less) dense as they propagate through the frontal zone between two water masses. The slope angle of the intrusions relative to the horizontal plane exceeds the slope angle of the isopycnals. The estimate of the hydrological parameters presented in [1, 2] showed that the front recorded at stations 40–42 is strongly baroclinic with the dominating contribution of the temperature to the mean vertical distribution of the density

Calculation of the parameters of intrusive layering for a baroclinic front

First of all, we decided to calculate the versions when the input parameters of the problem that determine the possibility of instability appearance exceed the real values of the corresponding nydrological parameters estimated from the empirical analysis [4]. The results of the calculation of the most unstable mode parameters such as the growth rate λ , vertical scale h, and slope relative to the horizontal plane are given in **Table 2**.

Table 3 presents the results of calculations for the cases when the hydrological parameters satisfactorily correspond to the parameters estimated from the empirical data:

$$\overline{T_x} \approx 0.5 \cdot 10^{-6} \, {}^{o}\mathrm{C} \, /\mathrm{m}, \, \overline{\varepsilon_z} = \frac{g \alpha T_z}{N^2} \approx 5 \, / \, 6, \, \overline{\varepsilon_x} = \frac{g \overline{\rho_x}}{N^2} = \overline{\gamma_\rho} \approx 4.2 \cdot 10^{-4}, \, N \approx 0.002 \, \mathrm{s}^{-1}$$

It is seen from Table 4 that, in two cases, the vertical scales of the most unstable mode and its slopes satisfactory correspond to the parameters of the observed intrusions, but the formation time of the unstable modes is greater than 10 years.

In the final stage, we decided to perform model calculations at the hydrological parameters of the problem, which differ from the parameters estimated from the empirical data but are close to them. The goal of these calculations was to simulate unstable modes whose parameters mostly correspond to the parameters of the observed intrusions. In this case, our efforts were aimed to select such input parameters of the problem at which restrictions on the coefficient of momentum diffusivity and the formation time of the most unstable modes would be fulfilled. The results are presented at Table 4.

It is seen from the data in the tables that the scale of the most unstable modes can range from 5 to 7 meters, i.e., approximately two-three times smaller than the scales of the observed intrusions. Thus, under specific restrictions on the coefficient of vertical momentum diffusivity, it is not possible to get good agreement between all the parameters of the unstable modes and observed intrusions in contrast to the modeling of the instability of a pure thermohaline front.