A possible mechanism of interleaving at weak baroclinic fronts under stable-stable stratification in the Arctic Basin

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Introduction

In the Eurasian Basin of the Arctic Ocean there are baroclinic and thermohaline fronts within the upper layer of the Polar Deep Water (PDW) populated with intrusive layers of vertical length scale as large as 30 m and horizontal scale reaching more than 100 km (Rudels et al., 1999, 2009; Kuzmina et al., 2011) observed at stable-stable stratification (i.e. when the mean salinity increases with depth while the mean temperature decreases with depth). It can be suggested that thermohaline intrusions within the upper layer of PDW are driven by differential mixing.

Merryfield (2002) was the first to show satisfactory agreement between calculations of unstable modes from a 3D interleaving model, taking into account differential mixing at a no baroclinicity front and observations of intrusive layering at a pure thermohaline front in the PDW. Merryfield's (2002) findings were confirmed by Kuzmina et al. (2014) for a pure thermohaline fronts. However, the 2D model of interleaving driven by differential mixing at the baroclinic front did not show a satisfactory fit simultaneously between the three modelled parameters, namely the vertical scale, growth time and slope of the fastest growing mode, and observations of intrusions in a frontal zone with substantial baroclinicity in the upper PDW layer (Kuzmina et al., 2014).

- Therefore, new approaches to the mathematical description of the formation of large intrusions in areas of baroclinic fronts appear relevant.
- We suggest that interleaving at baroclinic fronts may be considered as a result of 3D instability of weak geostrophic current due to combined effect of vertical linear shear and diffusion of density (buoyancy).

Effect of vertical diffusion of buoyancy on baroclinic instability of geostrophic zonal wind was studied theoretically by Miles (Miles, 1965). Based on an analogy between the equations describing dynamics of large-scale atmospheric perturbations and the Orr-Sommerfeld equation (Lin, 1955), Miles analyzed the instability of the critical layer (very thin layer in which the phase velocity of disturbance equals to the velocity of zonal flow). As a result, Miles built analytical asymptotic solution taking into account the very small, but finite vertical diffusion of buoyancy. Based on the analysis, Miles concluded that the influence of vertical diffusion of buoyancy to destabilise the zonal wind is not essential in comparison with baroclinic instability (generation of cyclones and anticyclones) for typical atmospheric geostrophic wind.

One can assume, however, that other situations can be observed in the deep ocean. Indeed, in the Polar zones, for example, in the Eurasian Basin of the Arctic, very weak geostrophic currents are observed at deep layers (Aagaard, 1981). These currents can have large horizontal (transverse) scale and large time scale, the latter is estimated at much more than one year (Aagaard, 1981). Taking into account that the influence of beta-effect on the dynamics of large-scale disturbances is negligible in the Polar Ocean, it seems reasonable to suggest that the role of diffusion of buoyancy in destabilization of weak geostrophic currents can be important. Therefore, in such circumstances one would expect the formation of intrusions, rather than vortices.

Problem formulation, derivation of basic equation and solutions

The equation of evolution of potential vorticity in the quasi-geostrophic approximation is:

$$\left(\frac{\partial}{\partial t}+\frac{U\partial}{\partial x}\right)\left(\frac{\partial^2 p}{\partial z^2}+\frac{N^2\Delta p}{f^2}\right)+\frac{\beta v N^2}{f}-vsf=\frac{K\partial^4 p}{\partial z^4}+\left(\frac{K' N^2}{f^2}\right)\left(\frac{\partial^2\Delta p}{\partial z^2}\right), \qquad (1)$$

where U is zonal component of the geostrophic velocity, N, f are the buoyancy frequency, Coriolis parameter, v is velocity fluctuation along the y axis, p is pressure fluctuation, $\beta = df/dy,$ and the x, y and z axes are directed eastward, northward and upward, respectively. The constant coefficients K and K' are treated as average values in an ocean layer under investigation.

We will consider at $K \approx K'$ long-wave disturbances (perturbation of a planetary scale) of weak geostrophic current which satisfy the following relationship between the vertical and horizontal length scales (H and L, respectively): $L \gg L_R$, where $L_R = NH/f$ is the baroclinic Rossby radius of deformation. If we describe the motion in the Arctic Basin, the β-effect term can be ignored because in the vicinity of the North Pole.

Taking into account the above mentioned conditions we may use the method of series expansion at small parameter $Bu = N^2H^2/f^2L^2$ (where is the Burger-number). At $Bu{\sim}10^{-3}-10^{-4}$ it is reasonable to consider only the first term of the series. In this case we can rewrite the potential vorticity equation in the simplified form:

$$\left(\frac{\partial}{\partial t}+U\partial/\partial x\right)(\partial p/\partial z)-(\partial p/\partial x)sz=K\partial^3 p/\partial z^3 \, . \qquad (2)$$

To analyse the instability of geostrophic flow, let us consider a layer of vertical scale of $2H_0$ and place the co-ordinate system on the middle line of the layer. For the analysis of the instability in the frame of equations (2) we will consider a geostrophic flow symmetric relative to the midline of the layer with quadratic z-dependence of velocity: $U=\frac{sz^2}{2}+U_3$, $U_3 = const.$

Spectral form (2):

$$ik\left(\frac{sz^2}{2}+U_3-c\right)\left(\frac{dF(z)}{dz}\right)-F(z)iksz=Kd^3F(z)/dz^3\ ,\ \ p=F(z)e^{ik(x-ct)}\sin\left(\frac{\pi y}{L}\right). \qquad (3)$$

It is logical to take the conditions of zero buoyancy flux (for density perturbations) at the layer boundaries : $p_{zz}=F(z)_{zz}=0$ at $z=\pm H_0$ (the type 1 boundary conditions). It is reasonable to consider another type of condition too, namely, the slippery boundary conditions or equivalent conditions of the vanishing density disturbances at the boundaries: $\frac{dv}{dz}=\frac{du}{dz}=\rho=0$ at $z=\pm H_0$ (the type 2 boundary conditions). Under the type 2 boundary conditions, it is necessary to require the absence of convergence or divergence of buoyancy flux within the layer: $p_{zz}(z=H_0)=p_{zz}(z=-H_0)$.

We found analytical solutions of the equation (3). Some examples of unstable ($Im\, c>0$) and stable ($Im\, c<0$) solutions are presented in the figures 1, 2 and 3.

Obtained solutions: comment

The unstable modes described by obtained solutions cannot be attributed to the critical layer instability .

Application to thermohaline intrusions in the Eurasian Basin of the Arctic Ocean

Using the model, we can obtain that the time formation of the unstable mode with the vertical scale ~ 40m is estimated as~ 5 years at $K=10^{-6}$ m²s⁻¹ and approximately 1.7 years at $K=3\cdot10^{-6}$ m²s⁻¹. These estimates of the formation time of intrusions in PDW are evidently better than the evaluations that can be obtained from 2D modelling of baroclinic front instability (Kuzmina et al., 2014).

Short conclusions (or Highlights)

- Combined effect of vertical linear shear and diffusion of density can cause 3D destabilisation of gestrophic flow
- General analytical solutions are found in quasi-geostrophic long-wave approximation
- A new mechanism of large-scale intrusions generation at stable-stable stratification is suggested.The proposed description of intrusions can be considered as a possible alternative to the mechanism of interleaving due to the differential mixing
- The unstable solutions are used to treat intrusions in the Arctic Ocean
- The stable long-wave disturbances unlike Rossby waves can move both to West and East
- Linear shear of the mean flow acts upon disturbances likewise the β-effect

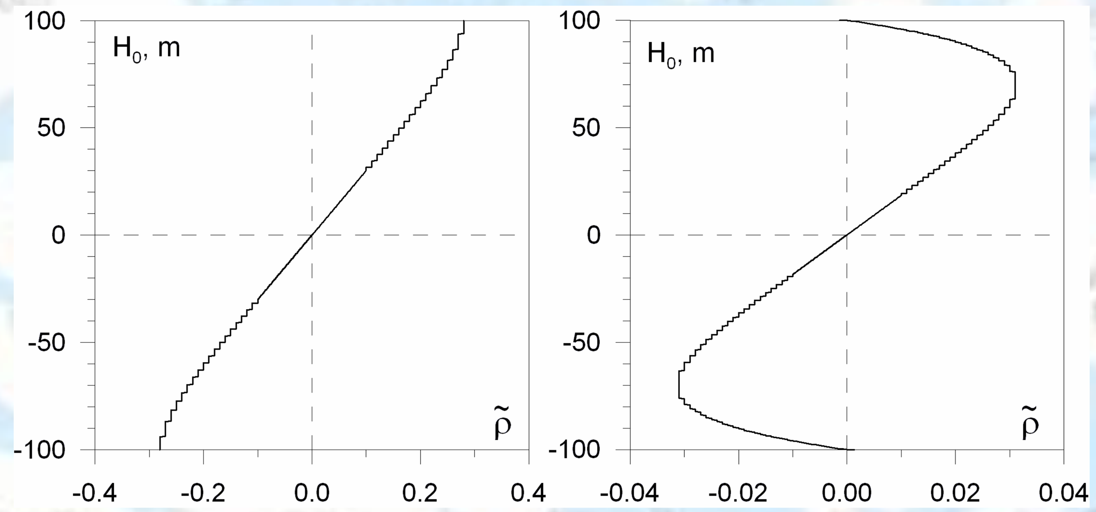

Fig. 1. Modelled vertical profiles of density disturbances $Re(dF/dz)=Re\rho=\tilde{\rho}$. Unstable solution increasing with $|z|\rightarrow\pm\infty$ for boundary conditions of type 1 (left) and type 2 (right) and typical hydrological parameters for the Arctic ocean.

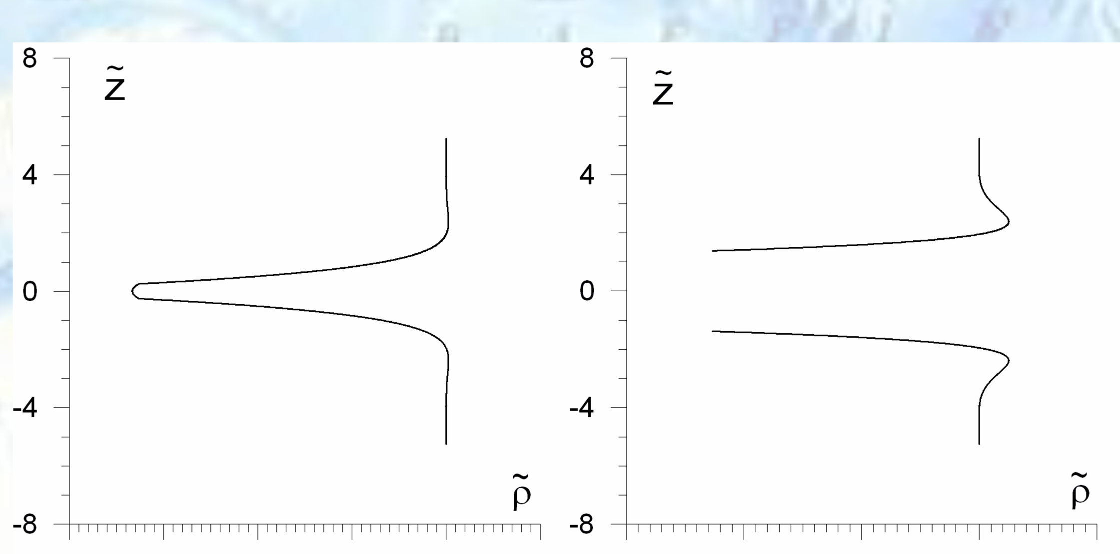

Fig. 2. Modelled vertical profile of density disturbances $Re(dF/dz)=Re\rho=\tilde{\rho}$. Unstable solution decreasing with $|z|\rightarrow\pm\infty$ (left) and fragment of this solution (right) depending on dimensionless co-ordinate $\tilde{z}=z\cdot(ks/K)^{1/4}$and typical hydrological parameters for the Arctic ocean.

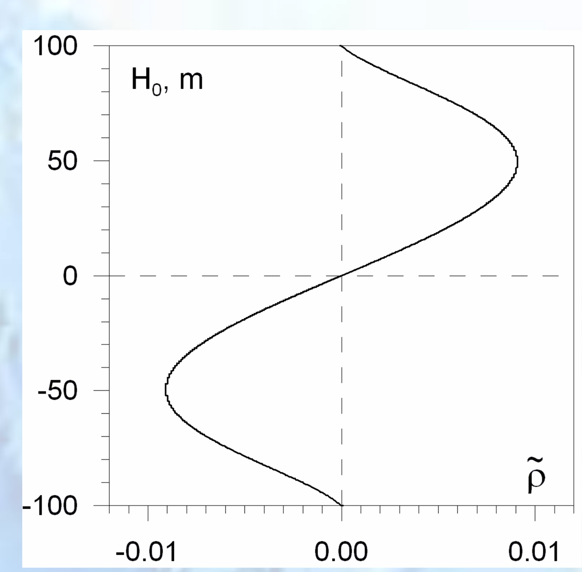

Fig. 3. The same as Fig. 1 but for stable solution for boundary conditions of type 2.